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Modeling & dynamic studies of heat transfer cooling of liquid in half-coil jackets

M.T. Dhotre *, Z.V.P. Murthy, N.S Jayakumar

D. D. Institute of Technology, Gujarat University, Nadiad 387001, India

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Abstract

The half-coil jacketed vessels find an extensive use in the chemical process industries for the process heating and cooling. The experimental studies of cooling of liquid using half-coil jacket is reported in the present work. The experiments are carried out for flow of coolant through half-coil jacket under both laminar and turbulent flow conditions. Correlations have been developed for the heat-transfer coefficient for the half coil jacket.

A mathematical model for the half-coil jacketed system is proposed by developing the differential energy balance equations for both shell and coil side. In the model, the coil side is divided into the definite number (n) of mixing sections. The value of n is found out through dimensionless variance obtained from the transient cooling water temperature data and is equal to three for laminar and one for turbulent flow conditions. The equations were solved by semi-Implicit Euler method to predict the shell side and coil side temperature. It is found that the model predictions are in satisfactory agreement with the experimental data.

Keywords: Mathematical model; Half-coil jacketed vessels; Heat transfer coefficient

1. Introduction

A large number of the industrial processes, such as the production of polymers, specialty and fine chemicals, pharmaceuticals, bio-products, as well as other products for which continuous production is not feasible, are operated batch wise. Especially in fine chemical or pharmaceutical industry, the batch or fed-batch reactor functions as the heart of the process transformation. In many cases this mode of operation is used to manufacture a variety of products that need significantly different characteristics such as conversion time, heat of reaction, etc. Due to the complexity of the reaction mixture and the difficulty to perform on-line composition measurements, control of batch reactors is essentially a problem of temperature control. Agitated vessels are generally used for processing liquid systems and are suitable for carrying out reaction at isothermal conditions where heat of reaction is high. Jacketed process vessels can be used when a close control of temperature is important, generally when chemical reaction takes place. Unwanted temperature fluctuations can degrade product quality in production of pharmaceuticals and special chemicals, improper temperature can be particularly costly. Generally, heating or cooling is affected by jacketing the vessel and the control performances are mainly dependent on the heating–cooling system associated with the reactor [1]. Jackets of several types like conventional jacket, plain jacket, dimple jacket, spiral jacket, channel jacket, half-coil jacket are available. Half coil jackets are comparatively more advantageous than others, as they offer high heat transfer coefficient on the half coil side because of proper circulation of the heat transfer fluid through the coil; parallel heating and cooling circuits can be provided through separate limpet circuits enabling use of different heating and cooling media; close temperature control can be obtained by having a small inventory of the heat transfer fluid in the limpets, which reduces the control response time significantly [2–4]; since half-coils are made of less material than conventional jackets, this is an economical way of jacketing. The design is ideal for hot oil applications because of the equipment high structural strength. It can be effectively used with water and in many cases is better than conventional

* Corresponding author. Present address: Nuclear Engineering and Safety Department, Laboratory of Thermal Hydraulics, Paul Scherrer Institute, CH-5232 Villigen-PSI, Switzerland. Tel.: +41 56 3102707; fax: +41 56 3102684.

E-mail address: mahesh_dhotre@yahoo.co.in (M.T. Dhotre).

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Nomenclature

A_i	inside heat transfer area (m^2)
A_o	outside heat transfer area on coil side (m^2)
A_w	logarithmic area of the wall (m^2)
C_{pc}	specific heat of cold fluid (J/kg K)
C_{ph}	specific heat of cold fluid (J/kg K)
d	diameter of the half-coil (m)
D_{eq}	equivalent diameter for the coil side (m)
h_i	inside film heat transfer coefficient (W/m^2 K)
h_o	outside film heat transfer coefficient (W/m^2 K)
k	thermal conductivity of the coolant (W/m K)
Nu_e	experimental Nusselt number)
p	pitch (m)
t	time (min)
T_c	temperature of cold liquid (K)
T_{co}	outlet temperature of cold liquid (K)
T_{ci}	inlet temperature of cold liquid (K)
$T_{c1}, T_{c2}, \ldots T_{cn-1}, T_n$	temperature of cold liquid coming through section 1, 2, ..., $n-1$, n (K)
T_h	temperature of hot liquid (K)
U_{oe}	time averaged overall heat transfer coefficient (W/m^2 K)
U_{op}	predicted overall heat transfer coefficient (W/m^2 K)
U_i	inside overall heat transfer coefficient (W/m^2 K)
U_o	outside overall heat transfer coefficient (W/m^2 K)
$(\Delta T)_{ln}$	logarithmic mean temperature difference (K)
V	volume of the shell side (m^3)
$V_1, V_2, \ldots, V_n$	volume of the section 1, 2, ..., to n (m^3)
q	flow rate of the cooling liquid (m^3/s)
X_w	thickness (m)

Greek letters

ρ_h	density of hot liquid (kg/m^3)
ρ_c	density of cold liquid (kg/m^3)
σ_θ^2	variance

jackets. Further, the half-coil jacket can be divided into multiple zones for maximum flexibility and efficiency, as there are no limitations to the number and location of inlet and outlet connections. Thus, either the entire jacket may be used or only as much as needed, so that various size batches can be processed economically in the same vessel. Multiple zoning reduces the pressure drop of the heat transfer medium in the jacket [5–8].

Although half-coil jackets are being used in the chemical process industries, hardly any experimental data of half-coil jacket has been reported and there exists practically no correlation or design model to our knowledge. In view of this, it was thought desirable to carry out the experimental and theoretical investigation for half-coil jackets. A mathematical model has been developed for the prediction of the heat transfer.

2. Theory

The individual and overall heat transfer coefficients were calculated from the heat balance with following assumptions:

- heat transfer resistance due to the wall of the vessel can be neglected;
- heat losses form vessel, lid, bottom and sides can be neglected;
- the heat accumulation on the vessel components (walls, stirrer, top, etc.) can be neglected;
- at any time, the temperature is uniform throughout the liquid inside the vessel;
- the coils are completely filled;
- the reactor dynamics are not considered, i.e. the heat transferred is instantly removed.

The following heat balance equations were used for evaluation of the average heat transfer coefficient. Heat balance on the shell side is given by:

$$-V_s \rho_h c_{ph} \left(\frac{dT_h}{dt} \right) = h_i A_i (\Delta T)_{ln} \tag{1}$$

where,

$$(\Delta T)_{ln} = \frac{(T_h - T_{ci}) - (T_h - T_{co})}{\ln(T_h - T_{ci}/T_h - T_{co})} \tag{2}$$

Heat balance on the half coil side is given by:

$$-q_c \rho_c c_{pc} (T_{co} - T_{ci}) = h_o A_o (\Delta T)_{ln} \tag{3}$$

where $(\Delta T)_{ln}$ is the logarithmic mean temperature difference. The properties ρ_c and c_{pc} were taken at the average temperature of T_{ci} and T_{co} for any single experiment. The properties ρ_h and c_{ph} were taken at the shell liquid temperature T_h at any time t. The experimental overall heat transfer coefficient based on outside area, U_{oe}, was obtained from the individual heat transfer coefficient as:

$$\left(\frac{1}{U_{oe} A_o} \right) = \left(\frac{1}{h_i A_i} \right) + \left(\frac{x_w}{k_w A_w} \right) + \left(\frac{1}{h_o A_o} \right) \tag{4}$$

where A_w is the logarithmic mean area of the wall. As the present work involves batch heat transfer study where individual and overall heat transfer vary with time, time averaged overall heat transfer coefficient calculated for each experiment as:

$$\bar{U}_{oe} = \frac{\sum_{J=1}^{M} U_{oe} t_J \Delta t_J}{\sum_{J=1}^{M} t_J \Delta t_J} \tag{5}$$

where M is the total number of readings in each experiment.

As half-coil was used in the present study, an equivalent diameter from the coil side has been defined which can be calculated from hydraulic mean radius as:

$$D_{eq} = 4 r_H = \frac{4(\pi d^2/8)}{(\pi d/2) + d} = \frac{\pi}{\pi + 2} d \tag{6}$$

Using the equivalent diameter, the average overall dimensionless heat transfer coefficient was calculated by the following expression:

$$\mathrm{Nu} = \frac{\bar{U}_{\mathrm{oe}} D_{\mathrm{eq}}}{k} \tag{7}$$

The conceivable factors affecting the Nusselt number are the liquid specific heat, the liquid thermal conductivity, density, and viscosity of the liquid. The least square method were applied to all the experimental data. The final correlation for the Nusselt number was found to be of the form:

$$\mathrm{Nu_e} = a\,\mathrm{De}^{b} \tag{8}$$

where Dean number is defined as:

$$\mathrm{De} = \left(\frac{D_{\mathrm{eq}}\rho_{\mathrm{c}} V}{\mu_{\mathrm{c}}}\right)\left(\frac{D_{\mathrm{eq}}}{d_{\mathrm{c}}}\right)^{0.5} \tag{9}$$

and parameters a and b are constant can be calculated using the experimental data. The ρ_{c} and μ_{c} are evaluated at the arithmetic mean temperature of initial cooling water temperature at the inlet and final cooling water temperature at the outlet at the end of the experiment.

3. Heat transfer model

A mathematical model for the half-coil jacket system has been developed using differential energy balance equations for both shell side as well as jacket side. The jacket volume has been divided into the n parts and the differential energy balance equations for both shell as well as jacket side. Each volume is assumed to be mixed homogeneous system wherein temperature will not vary in the volume. Fig. 1 represents schematic model for the half-coil jacket.

Heat balance on Section 1 is given by:

$$-V_1\rho_{\mathrm{c}}c_{\mathrm{PC}}\left(\frac{\mathrm{d}T_{\mathrm{c1}}}{\mathrm{d}t}\right) = q\rho_{\mathrm{c}}c_{\mathrm{pc}}(T_{\mathrm{ci}} - T_{\mathrm{c1}}) + U_{\mathrm{op}}(A/n)(\Delta T)_{\ln 1} \tag{10}$$

where $(\Delta T)_{\ln 1}$ is logarithmic temperature difference for Section 1 given by:

$$(\Delta T)_{\ln 1} = \frac{(T - T_{\mathrm{ci}}) - (T - T_{\mathrm{c1}})}{\ln(T - T_{\mathrm{ci}}/T - T_{\mathrm{c1}})} \tag{11}$$

Heat balance on Section 2:

$$-V_2\rho_{\mathrm{c}}c_{\mathrm{pc}}\left(\frac{\mathrm{d}T_{\mathrm{c2}}}{\mathrm{d}t}\right) = q\rho_{\mathrm{c}}c_{\mathrm{pc}}(T_{\mathrm{c1}} - T_{\mathrm{c2}}) + U_{\mathrm{op}}(A/n)(\Delta T)_{\ln 2} \tag{12}$$

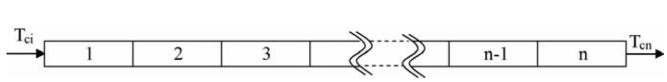

Fig. 1. Schematic of the representation of the model.

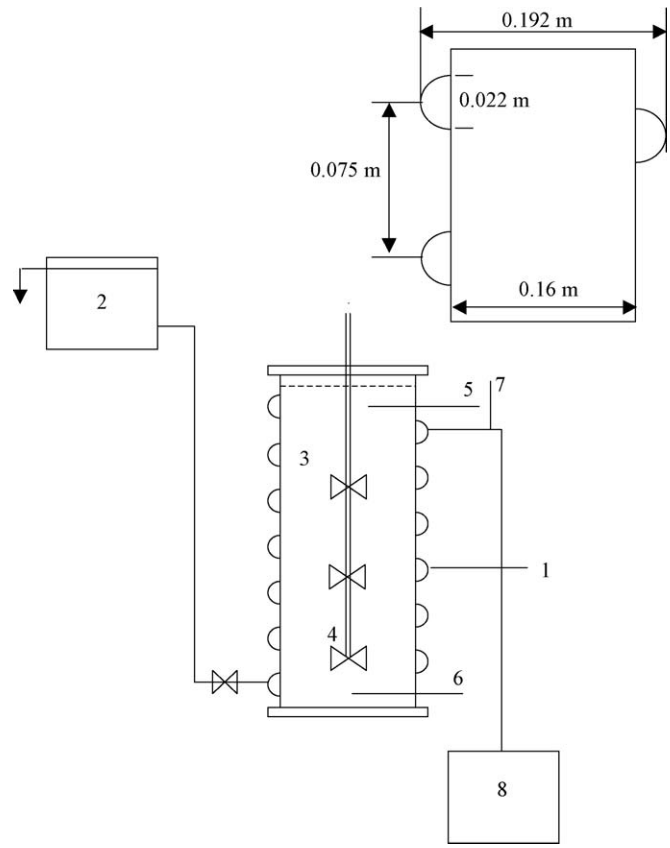

Fig. 2. Experimental set-up for half-coil jacket. (1) Half coil jacke. (2) Overhead tank. (3) Reactor. (4) Impeller. (5) and (6) thermometers. (7) Thermometer in cooling water. (8) Collector.

where $(\Delta T)_{\ln 2}$ is logarithmic temperature difference for Section 2 given by:

$$(\Delta T)_{\ln 1} = \frac{(T - T_{\mathrm{c1}}) - (T - T_{\mathrm{c2}})}{\ln(T - T_{\mathrm{c1}}/T - T_{\mathrm{c2}})} \tag{13}$$

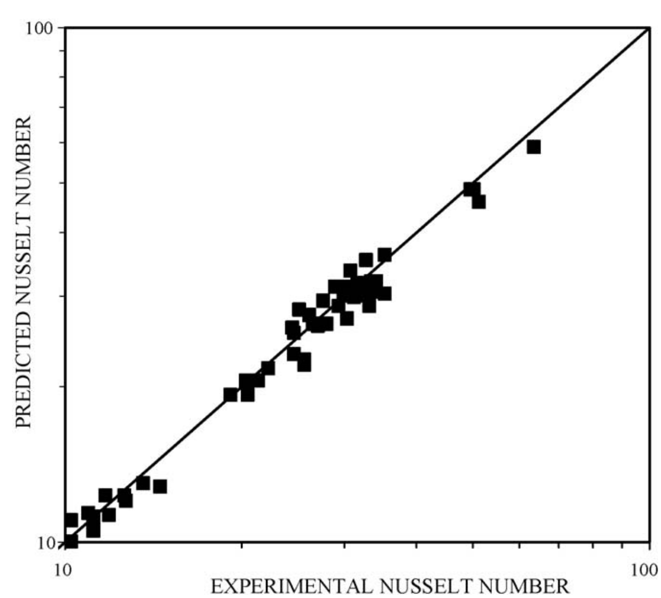

Fig. 3. The parity plot of the experimental and predicted Nusselt number.

Similarly, heat balance on nth section is given by:

$$-V_{\mathrm{n}}\rho_{\mathrm{c}}c_{\mathrm{pc}}\left(\frac{\mathrm{d}T_{\mathrm{cn}}}{\mathrm{d}t}\right)=q\rho_{\mathrm{c}}c_{\mathrm{pc}}(T_{\mathrm{cn-1}}-T_{\mathrm{cn}})+U_{\mathrm{op}}(A/n)(\Delta T)_{\ln n} \tag{14}$$

where $(\Delta T)_{\ln n}$ is the logarithmic temperature difference for section n given by:

$$(\Delta T)_{\ln n}=\frac{(T-T_{\mathrm{cn-1}})-(T-T_{\mathrm{cn}})}{\ln(T-T_{\mathrm{cn-1}}/T-T_{\mathrm{cn}})} \tag{15}$$

The heat balance on shell side is given by:

$$-V\rho_{\mathrm{h}}c_{\mathrm{ph}}\left(\frac{\mathrm{d}T_{\mathrm{h}}}{\mathrm{d}t}\right)=U_{\mathrm{op}}A_{\mathrm{o}}(\Delta T)_{\ln} \tag{16}$$

where $(\Delta T)_{\ln}$ is logarithmic temperature difference for the shell given by:

$$(\Delta T)_{\ln}=\frac{(T_{\mathrm{h}}-T_{\mathrm{ci}})-(T_{\mathrm{h}}-T_{\mathrm{cn}})}{\ln(T_{\mathrm{h}}-T_{\mathrm{ci}}/T_{\mathrm{h}}-T_{\mathrm{cn}})} \tag{17}$$

Tank in series model for the jacket is an alternate approach to dispersion model for dealing with deviation from plug flow. In this model, we assume the outlet jacket section can be represented by a series of n equal size back mixed flow vessels. The number of completely mixed section, n, for the jacket volume can be obtained by calculating dimensionless variance (σ_θ^2) using the transient cooling water data.

$$\sigma_\theta^2=\frac{\sum_{i=1}^{M}(t-\bar{t})^2E(t)\Delta t}{\bar{t}^2} \tag{18}$$

where

$$E(t)=\frac{T_{\mathrm{co}}(t)}{\sum_{i=1}^{M}T_{\mathrm{co}}(t)\Delta t} \tag{19}$$

$$\bar{t}=\frac{\sum_{i=1}^{M}tT_{\mathrm{co}}(t)}{\sum_{i=1}^{M}T_{\mathrm{co}}(t)} \tag{20}$$

Number of completely mixed flow vessels, n can be obtained from following expression:

$$n=\frac{1}{\sigma_\theta^2} \tag{21}$$

The value of n from above equation is used for solving the heat transfer model Eqs. (10)–(17) numerically to predict the shell side and coil side temperature. The value of U_{op} was calculated using correlation.

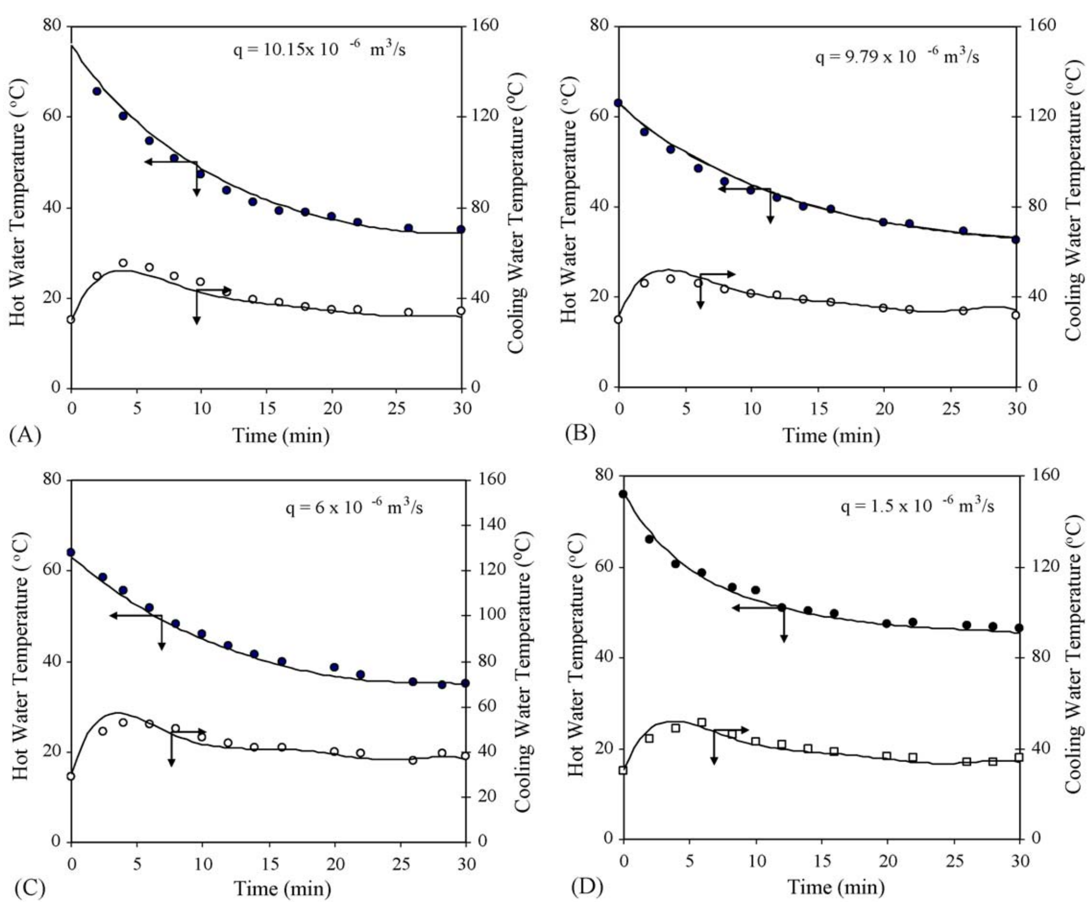

Fig. 4. Comparison of the experimental data and predicted temperature data for laminar flow: (A) 10×10^{-6} m^3/s; (B) 8×10^{-6} m^3/s; (C) 6×10^{-6} m^3/s; (D) 1.5×10^{-6} m^3/s.

Table 1
Experimental conditions

Vessel geometry	
Diameter, D (m)	0.16
Thickness, X_W (m)	0.005
Height (m)	0.58
Coil geometry	
Diameter of the limpet tube, d (m)	0.02
Equivalent diameter of half-coil, D_{eq} (m)	0.01344
Diameter of coil, D_c (m)	0.192
Length of coil, L (m)	3.25/2.65
Pitch, p (m)	0.075
Number of turns	4 and 6
Dimensionless numbers	
Dean number	50–4000
Reynolds number	400–15000
Nusselt number	5–45

4. Experimental

The schematic diagram of the apparatus used for the present investigation is shown in Fig. 2. It consisted of a reaction vessel of 0.16 m i.d. which is made up of mild steel. The vessel was jacketed by half-coils. The pipe used for the half-coil is of 0.022 m i.d. and of 3.25 m length. The outside portion of vessel was insulated by glass wool while top and bottom was insulated using thermocole. A double flat blade stirrer was used inside the vessel to ensure uniform temperature at any time. The speed of the stirrer was kept constant for all the experiments. The vessel was filled with the liquid to an appropriate depth with known initial temperature. The cooling water with known initial temperature and flow rate were circulated through half coils. The temperature versus time data was collected from both vessel as well as coil side using the thermocouples. The experiments were carried out in the similar manner at different flow rates of cooling water on the coil side. The temperature of hot water and coolant at different time interval were read from the measured data and used for calculating logarithmic mean temperature difference and to find out the thermal properties. Table 1 gives the experimental conditions for all the runs.

5. Result and discussion

5.1. Proposed correlation

A dimensionless correlation relating the time averaged overall heat transfer coefficient was found be function of Dean number for both laminar and turbulent flow conditions. On the

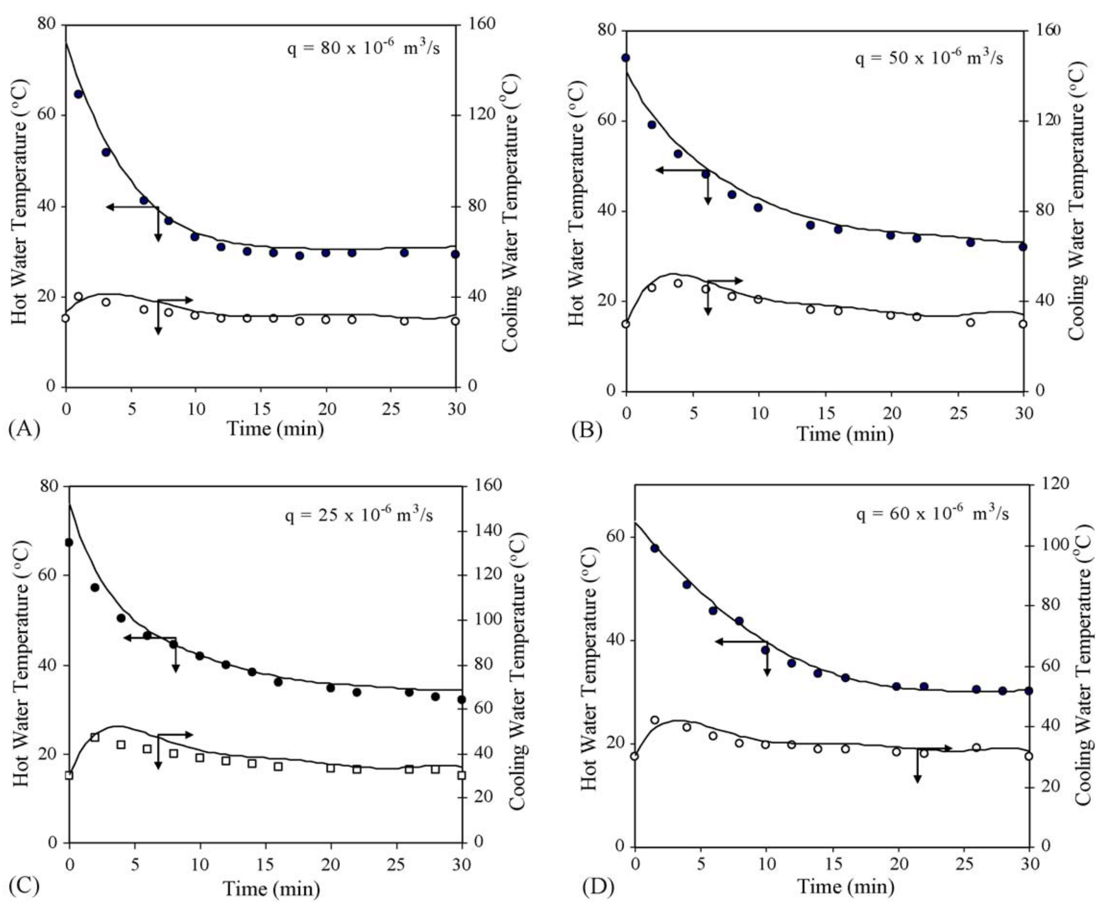

Fig. 5. Comparison of the experimental data and predicted temperature data for turbulent flow: (A) 80×10^{-6} m^3/s; (B) 60×10^{-6} m^3/s; (C) 50×10^{-6} m^3/s; (D) 25×10^{-6} m^3/s.

basis of reported experimental data following correlation has been proposed:

For laminar flow:

$$\mathrm{Nu} = 0.788(\mathrm{De})^{0.6} \tag{22a}$$

For turbulent flow:

$$\mathrm{Nu} = 0.21(\mathrm{De})^{0.7} \tag{22b}$$

The comparison of the experimental and the predicted Nusselt number by Eqs. (8), (22a) and (22b) is shown in Fig. 3. It can be seen that the fairly good agreement between the experimental and the predicted Nusselt number.

5.2. Proposed mathematical model

The predicted average value of overall heat transfer coefficient from Eqs. (22a) and (22b) was used in the mathematical model to predict the shell and coil side temperatures. The mathematical, model predicts the shell side and coil side temperature. Simulations were carried out for both laminar and turbulent conditions. The value of n was found to be 3 and 1 for laminar and turbulent condition. Figs. 4 and 5 shows the comparison of shell side and coil side temperature with the model predictions for laminar and turbulent case. It can be seen that the predictions are in excellent agreement with the experimental data.

6. Conclusion

Heat transfer cooling of liquid has been studied experimentally and two empirical correlations were proposed for prediction of overall heat transfer coefficient for flow coolant liquid water through half-coil jacket under both laminar and turbulent conditions.

A mathematical model for the half-coil jacketed system has been proposed by developing the differential energy balance equation for both shell side and coil side. The model predictions were found to be in good agreement with the experimental data.

The simple model presented here can be of great use to the practicing engineer.

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